Stat 201: Introduction to Statistics

Standard 15: Probability – Law of Large Numbers

From *Naked Statistics: Probability*

- "Why are these companies willing to assume such risks? Because they will earn large profits in the **long run** if they price their premiums correctly. Obviously, some cars insured by Allstate will get stolen. Others will get totaled when their owners drive over a fire hydrant,… but most cars insured by Allstate or any other company will be just fine."

Law of Large Numbers

- (**LLN 1**) – As the sample size increases the sample estimates ($\bar{x}$ or $\hat{p}$) approach the population values (μ or ρ)

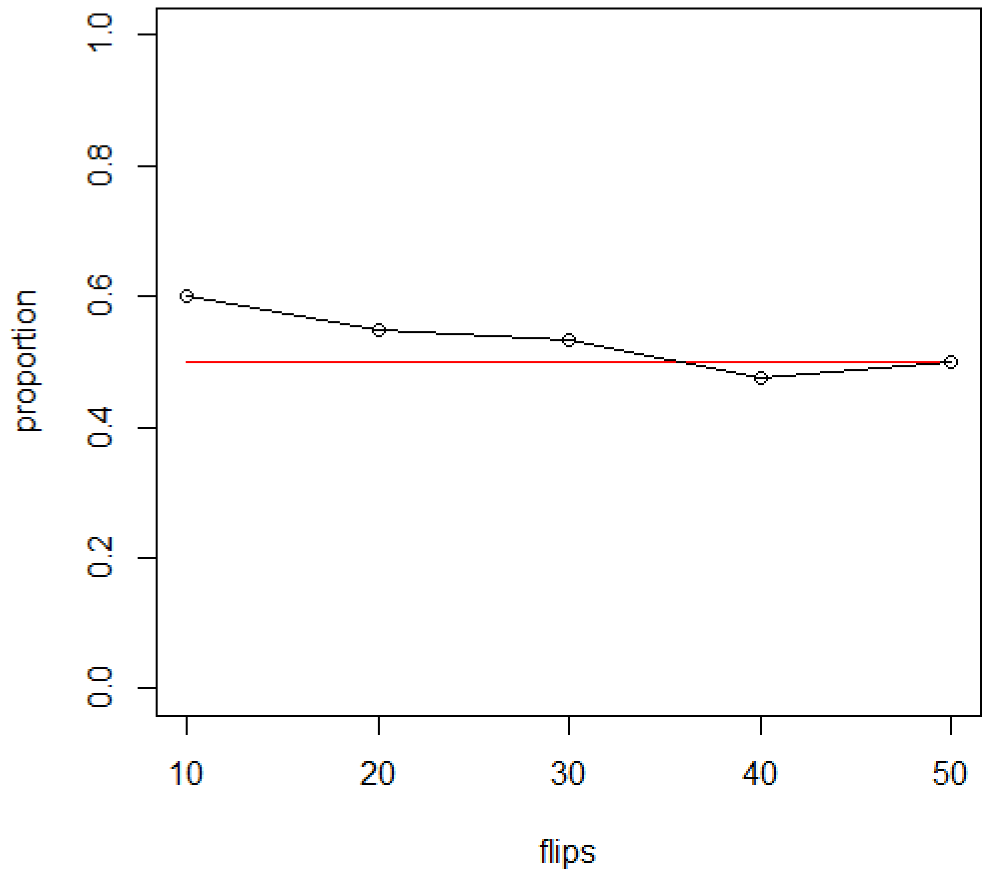
- (**LLN 2**) – As the number of trials increase the **proportion** of occurrences of any given outcome approaches the probability in the long run. (This is seen by $\hat{p}$ → ρ above)

Simulation of Coin Flips

- 10 flips: 6 heads were flipped
  - $Total\ proportion\ \hat{p}\ = \frac{x}{n} = \frac{6}{10} = .60 = 60\%\ heads$
- 10 more flips: 5 heads were flipped
  - $Total\ proportion\ \hat{p} = \frac{x}{n} = \frac{5+6}{10+10} = \frac{11}{20} = .55 = 55\%\ heads$
- 10 more flips: 5 heads were flipped
  - $Total\ proportion\ \hat{p} = \frac{x}{n} = \frac{11+5}{20+10} = \frac{16}{30} = .5333 = 53.33\%\ heads$
- 10 more flips: 3 heads were flipped
  - $Total\ proportion\ \hat{p} = \frac{x}{n} = \frac{16+3}{30+10} = \frac{19}{40} = .475 = 47.5\%\ heads$
- 10 more flips: 6 heads were flipped
  - $Total\ proportion\ \widehat{p} = \frac{x}{n} = \frac{19+6}{40+10} = \frac{25}{50} = .5 = 50\%\ heads$

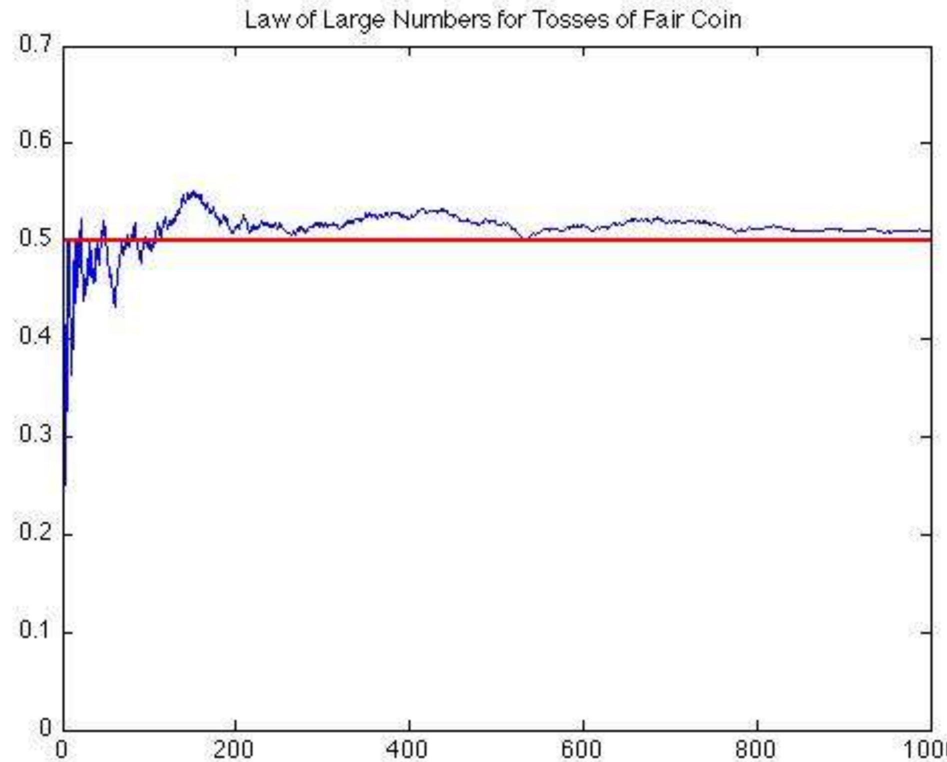
Simulation of Coin Flips

- (**LLN**) – As the number of flips increase the **proportion** of heads approaches the **probability** of seeing a heads, P(heads)=.5, which is the red line.



A Bigger Example of the Law of Large Numbers

- At first the proportion is all over the place – you can see the large spikes in the graph
- Importantly, we see that the proportion of coins that landed on heads levels off and gets closer and closer to 50%, the probability, which is where we expect it to go 'in the long run!'



Long-run Probability

- The probability of a particular outcome is the proportion of times that the outcome would occur in the long-run, as our sample size grows unbounded.
  - Note: the probability is probably the way you already think about it – you just never knew you were doing ‘long run’ probability!
  - This is how we justify using empirical probabilities!

Long-run Probability

- https://www.youtube.com/watch?v=MntX3zWNWec
- https://www.youtube.com/watch?v=UUlmDIi_JuY